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## FORMAL THOUGHT THE BASIS OF KENLORE.

EPISTEMOLOGY is one of the most ponderous words in the English language, but it can easily be replaced by the simple Saxon term "kenlore," which describes the process of cognition and would thereby explain how things come within the range of our ken. Our readers will find the term used in Professor Halsted's most interesting article on "The Unverifiable Hypotheses of Science," published in the present number of *The Monist*, where he rightly claims that there are certain hypotheses which "are yet neither confirmed nor refuted by the experiences which we possess or which we can hope to attain." One of them is the parallel postulate on which Eulidean geometry rests, and he is well aware of the fact that there are other geometries based upon other assumptions which are just as "unverifiable and irrefutable," and he calls them "okapi" hypotheses, comparing them to that rare giraffe species called *Ocapia Johnston's Sclater*. Professor Halsted's treatment, based on Poincaré's *Science and Hypothesis*, is quite instructive because he raises an important problem and ventilates it in his own ingenious way. We will add, however, that on several salient points we do not agree with him, and so we will here improve the opportunity of presenting our own views in contrast to his.

The difference between our views and those of Professor Halsted are perhaps insignificant when compared to the agreement between us. Like him we believe "that sci-

ence is to be the light of the world," and it may be that we only explain the facts on which he, together with Professor Poincaré, insists.

Professor Halsted enumerates his unverifiable hypotheses as, "(1) the existence of continuous extensive quanta in nature, (2) the principles of geometry, (3) the principles of mechanics and of the general theory of energy."

Professor Halsted distinguishes between unverifiable hypotheses and the real scientific hypotheses, saying that the former can neither be proved nor refuted while the latter are subject to verification. He overlooks, or at least tacitly passes by, the fact that these so-called unverifiable hypotheses are at the bottom of all scientific thought. They are, what Kant calls transcendental, the condition of cognition itself, and thus enter into every part of the fabric of our thought. For this reason mystics have actually claimed that all knowledge is unverifiable, and what Professor Halsted calls scientific hypotheses, which, he claims, are subject to verification, presuppose that the other more general problems of the unverifiable hypotheses have been settled. They refer to those special problems in which all interest is concentrated on the evaluation of definite facts of experience while the underlying general principles of thinking are taken for granted. Nowhere in our thought can we dispense with the general principles of the formal sciences.

Professor Halsted does not seem to be aware of the fact that all of his so-called unverifiable hypotheses refer to the underlying principles of the formal sciences, and they will be disposed of by a solution of the problem as to the nature of form and formal thought. We have discussed the problem of formal thought repeatedly and will here restate a summary of our solution, which takes a middle ground between the Kantian theory of the *a priori* and experimental sensualism as represented by Locke and his followers,

though we must add that upon the whole we follow Kant more than the sensualists because Kant saw the problem without solving it, while the sensualists attempted a solution without being familiar with the problem itself.

Briefly stated the situation is this. A sentient being becomes acquainted with the objective world through contact with surrounding things. Contact makes impressions which are felt by the senses and in their totality are called experience. The essential part of experience consists of feeling, and if we refer to the feeling alone we speak of it as sense experience but all the several experiences are of different forms, yea the variety of form is the most obtrusive difference by which the various objects are recognized. Every form makes an impression of its own. It makes its own path and is registered somewhere in the cerebral substance. A new impression follows the track into which it fits, and in awakening the memory traces which are like it, it is felt to be the same as they. This is the origin of representative thought, of concepts which have a meaning. In the higher stage of development a thinking being learns to make abstracts. He learns to take notice of qualities, and one of the most important qualities is the quality of form.

The term "form" as here used, implies not only external shape but also internal conformation or structure. It also comprises juxtaposition of parts or interrelations between things, and it is but natural that the highest abstract of this kind results in a conception of pure form, or the mere possibility of interrelations. This condition is produced by abstraction which empties our experience of all sensory elements, leaving only a potentiality for interrelational constructions, a void which in my *Foundations of Geometry* I have called "the field of anyness."

Here lies the foundation of mathematics. The mathematician does not start from nothing but from this void

which has been produced by clearing away all particularity, thus leaving a homogeneous field of universality.

It will be noticed that all the most important axioms of mathematics as well as the main principles of Aristotelian logic are unconsciously derived from this abstract notion of pure form. This void, this field of anyness, has been derived from the facts of experience, but the purely formal sciences are mere mental constructions.

The notion that the mathematician starts from nothing is wrong. The field of his operations, the void, has been abstracted from experience and possesses the potentiality of relationships from which particularities (matter and energy) are rigorously excluded. This confers by induction definite and positive qualities upon the void, which are homogeneity and universality. This means that a definite construction remains the same wherever and whenever it may be made, and it applies to any possible configuration of the same kind. This quality we have called "anyness," and it is obvious that this anyness of the formal sciences, being due to our own making, can neither be confirmed nor refuted by experience.

Properly speaking anyness is not an hypothesis. It is the result of a mental operation; it is a product of abstraction, and the data from which it has been derived have been furnished us by experience in the broadest sense of the word. Accordingly in one way the purely formal sciences are based upon experience, and in another way they are in the Kantian sense *a priori* constructions.

Experience is of an objective character, and thus the abstraction of pure form is ultimately derived from our notion of the objective world, yet the general idea of pure form furnishes only the condition for the construction of the formal sciences. The real execution of the work is done by operations of the mind and so this part is purely mental or subjective.

The strange thing about our purely formal thought is the fact that no knowledge is possible without it. It is the condition of any knowledge, and this feature of it is called by Kant transcendentalism. As all things are configurations, so the mind itself originates from relational functions by distinguishing between forms, by taking note of interrelations, by classifying types as genera and species, by tracing the interdependence of events, etc., and the principles of all these activities are identical with the principles of formal thought.

From these considerations it appears as a matter of course that being of a purely formal character the general principles of the purely formal sciences are, as Kant rightly recognized, the conditions of all experience, or, to use his term, they are transcendental. They can not be derived from the data furnished by the senses and so they can neither be confirmed nor refuted by experience.

The data of the senses are always particularities, but the fundamental principle of the formal sciences is the idea of universality, which does not exist in the world of concrete things. If we so please we may call the homogeneity of the void an assumption or, with Professor Halsted, an unverifiable hypothesis, but these names convey the idea that the principle of universality, the idea of anyness and whatever may be implied thereby are arbitrary notions, that they are unjustified and unjustifiable, while to any one who has followed our arguments this is obviously far from the mark, and even Professor Halsted grants that they are "not merely arbitrary conventions."

If we bear in mind our solution of the problem of form and formal thought, many difficulties which have puzzled scientists and philosophers and also mathematicians admit of an easy explanation. One of them is the problem of the irrational, another the problem of the infinitely great and the infinitely small, and we will in this connection refer

to our prior article in *The Monist* for January, 1910, on "The Nature of Logical and Mathematical Thought," especially the part written in reply to Prof. Bertrand Russell (pp. 46 ff.).

The objectionable feature of infinitude originates mainly through our neglect to regard mathematical magnitudes as functions. Every number is really the product of a function, and infinitude as well as zero are also numbers. All the zeros and all the infinitudes are not necessarily equal, for their value depends upon the function by which they have been produced. It would be a mistake to regard all infinitudes as the same regardless of their meaning and origin. By an infinitude we understand a function which can never be completed. If we consider infinitudes as objective things that have been completed we become involved in contradictions, by which we will naturally be puzzled, just as is the man who tries to catch his shadow, or the Chinaman who turns around to see his queue. The definition which declares that "an infinite aggregate is one equivalent to a part of itself" is an ingenious paradox, and we regard it as only relatively true. If it were taken seriously it would lead to mysticism.

Whether or not we can call the theorems of the formal sciences "*a priori* necessities" depends entirely upon our definitions both of *a priori* and necessity. If we accept Kant's meaning of those terms, purely formal theorems are certainly *a priori* necessities. Every one, if it has been correctly formulated, is on its own ground necessary. But we have called attention to the fact that there are certain degrees of *a priori*. There is the *a priori*, first, of logical, and then of geometrical constructions. The former is a static *a priori* referring to stationary relations among logical types; it is the *a priori* of being. The latter, a dynamic construction in which the possibility of pure motion is presupposed, is the *a priori* of doing.

The domain of the *a priori* can be more or less rigid, and when we come to the Euclidean system of homoloidal space in which two parallel straight lines will cut only if produced to infinity, we may construct other geometrical systems in which this principle of parallel lines does not hold. They are just as purely mental constructions as is Euclidean geometry, yet besides the giraffe there may also exist an okapi, and when we construct the geometry of the even plane we must grant that there are other geometries possible, such as the geometry of the sphere, of the pseudo-sphere, of four-dimensional bodies, of curved space, etc. If our systems of purely formal thought are mental constructions, it stands to reason that we ought to be able to create different systems based upon different principles.

We do not deny the possibility of other geometrical constructions than those of Euclid, but we regard them more as evidences of the fact that geometry is a mental construction, than that other assumptions will prove as useful or as applicable to reality. And so it is but natural that these variations have excited the curiosity of only scholarly minds who have little or no interest in practical affairs and delight in the idea that there are vast regions of possibilities which have never been opened up to this commercially minded generation. It can scarcely be denied that the systems of Bolyai and Lobatchevsky command a purely theoretical interest, and that so far as the pragmatic issues of life are concerned they may be regarded as still-born children of the genius of mathematics.

From this standpoint the name "okapi" which Professor Halsted has chosen for these theories seems very appropriate, for the okapi is an animal which is on the verge of extinction. The fate of this animal is tragic, for it has scarcely burst into fame when we know that it will soon disappear again.

For those who are not specialists in zoology, I will say

that the okapi has been known only since 1900. It lives in remote parts of the Congo Free State and being very shy it can be trapped by the natives only in pits. It is practically a stunted giraffe. Its forefeet are shorter and its neck less high than those of its more favored cousins. In place of horns it has mere buttons, and its skeleton most resembles those extinct primitive types of its species which from the places where they have been found, in Hellas and in Samos, have been named Helladotherium and Samotherium. Naturalists have become assured of the existence of this rare animal through its fur and skeleton, because it has been impossible to bring any living specimen within the sight of the white man. The probability is that the okapi will soon join the choir invisible where it will be in the company of the Helladotherium and the Samotherium.

As the okapi is a mere deviation from the giraffe type, so the okapi theories of mathematics are in principle like their better known cousin, Euclidean geometry. They are merely a variety which however proves less fit for survival. The okapi will have disappeared by the time civilization has reached to its present abode.

I learn to my surprise from Professor Halsted that the word "okapi" has only recently been introduced to the English speaking public in the new appendix of the *Century Dictionary*, whereas the continental lexicons have been familiar with it since the appearance of the first travelers' reports about the existence of this strange beast.

It is true enough that "the debate 'what is truth,' is a wrangle unless it give precedence to the kenlore question," but we would deny Professor Halsted's assumption that "we find our knowing is ever subject to our wishing." Reality intrudes upon us and we become acquainted with facts whether or not we wish to have any acquaintance with them, and their nature does not depend upon our desire.

The facts are that the thinking subject, being part of

reality, is in constant interaction with it, and the problem is, how did the thinking subject originate from reality and how is a representation of reality possible in the thinking subject? This, as we have stated, is answered through the formal sciences. The formal sciences are possible because the most significant attribute of reality is form, and having gained a general knowledge of pure form through abstraction from reality, the thinking subject constructs systems of pure form which, when we try to describe reality, can be used as methods of cognition for measuring, counting, and tracing interrelations.

Kant wondered why our notions of pure form should tally with the conditions of the objective world, of nature, of reality; but this fundamental problem of kenlore is solved if we bear in mind that the general notion of pure form has been derived through experience from the objective world.

Formal thought is the origin of cognition and it is applicable to objective existence because form is the essential feature of all things. It is not an accident that the natural laws (e. g., Kepler's and Newton's laws) are summed up in "formulas" and that universals of any kind are best described by the word "uniformities." Science practically consists of classifying forms, of noting interrelations and tracing transformations.

Even the law of the conservation of energy is based upon this same foundation. It is a purely formal statement, for it simply means that nothing originates and nothing is annihilated, all processes of nature are transformations. The law of conservation of matter and energy is as purely *a priori* as the propositions  $1 + 0 = 1$  and  $1 - 0 = 1$ . It can neither be refuted nor proved by experience, because the idea has not been derived from experience but is a product of mental reflection, the result of pure thought.

The same is true of causation which is only the positive

aspect of the law of transformation of which the law of conservation of energy is the negative counterpart. We can trace the concatenation of cause and effect, but we can never prove its universality from experience.

Form is a feature of reality and formal thought originates in thinking beings in response to the actions of the form-conditions of their surroundings with which they become acquainted by experience.

The systematic character of the formal sciences is of our own making, but the conditions of these mental constructions have been quarried out of the mine of experience, and so our conception of form is merely the picture of form in the objective world, as it is mirrored in the human mind.

We conclude this exposition of the part which ideal constructions play in kenlore by an example. We pin a silken thread down in two points and move the point of a pencil at its stretched end. The line resulting from this operation turns out to be an ellipse and ellipses are the paths of the planets. We study the nature of ellipses and formulate the theorems which we learn from the observation of our constructions and when we watch the motions of the planets in the heavens we can by the help of the geometry of ellipses predetermine the progress and all further positions of the planets.

It is a strange fact that these constructions made of ideas of pure form can be so helpful. They serve us as a key to experience, yea these purely formal thoughts are the mentality of our mind. They furnish the method by which sense impressions change into intelligible experience, and the elements from which they grow, being notions of pure form, can not be traced in the sense elements of experience.

The data of sense experience furnish particular facts but not the principle of universality. They are single items, but not the method according to which they appear as instances of general types. They are definite events or concrete

things, not laws, nor norms which would explain why they happen to be such as they are and what they would be like if in one way or another conditions were changed. The latter, laws, principles, methods, are mind-made; the former, the facts of experience, are nature, and nature is a play of transformations.

The laws of pure form are mind-made, but mind in its turn is nature-made. Sentient substance originates and nature impresses its own character into its fabric. There is first a sensing of concrete forms, then a recognition of pure form (that is to say, of form in abstract thought, of form in and by itself) and finally we have the methodical construction of systems of pure form.

The interrelations and interactions of feelings, their formal feature, is what we commonly designate as "mind"; and a systematized conception of them is in a word called "reason." There is no reason, no argumentative faculty such as is human reason, in nature, but there is a formative cosmic order determining all the particular facts of objective existence, and of this cosmic formative order human reason is an echo. From this norm which dominates the world and which is reconstructed in our mind we derive those principles of all our purely formal methods, our principles of logic and logical necessity, of universality, of our fundamental conditions for mathematical thought and geometrical constructions, and here accordingly lies the cornerstone of kenlore.

EDITOR.